CALCULATION OF THE MOTION OF A ROTOR IN A CYLINDRICAL CHAMBER

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The problem of the motion of an unbalanced rigid cylinder rotating in a stationary circular cylindrical chamber is solved numerically. The chamber is of finite length and is filled with a viscous gas. External forces which change periodically over time act on the inside surface of the cylinder. Calculations determine the steady trajectories of the rotor for different rotor velocities, the magnitude of the imbalance, and the amplitude and frequency of the external forces. The conditions for noncontact motion of the rotating cylinder in the chamber are determined.

1. We will examine two coaxial circular cylinders of length L and radii R_1 and R_2 (Fig. 1). The space between the cylinders is filled with a viscous gas. The center of mass of the rigid, solid internal cylinder is located outside its axis of rotation (static disequilibrium). The rotor turns about its own axis of symmetry with a constant angular velocity ω. The external cylinder (the chamber) is stationary. The gap between the cylinders is considerably smaller than their radii, so that the Reynolds equations can be used to find the pressure distribution in the thin layer of gas. The equation for pressure p has the following form [1] in a cylindrical coordinate system (r, ϕ, z) whose z axis is directed along the external cylinder

$$\frac{1}{12\mu}\frac{\partial}{\partial z}\left(h^3\rho\frac{\partial p}{\partial z}\right)+\frac{1}{12\mu R_1^2}\left(h^3\rho\frac{\partial p}{\partial\varphi}\right)=\frac{\partial}{\partial t}(\rho h)+\frac{\omega}{2}\frac{\partial}{\partial\varphi}(\rho h), \tag{1.1}$$

where ρ is the density of the gas; h = h(ϕ) is the local thickness of the gap between the cylindrical surfaces ($R_1 \le r \le R_1 + h$); μ is the absolute viscosity of the gas.

The boundary conditions:

$$\text{at } z=\pm L/2\; p=p_0 \tag{1.2}$$

(p_0 is the pressure in the medium surrounding the layer).

We use the pressure field in the gas layer that was found from problem (1.1)-(1.2) to determine the force applied to rotating cylinder of length L from the direction of the gas:

$$F_x=-2\int_0^{L/2}\int_0^{2\pi} pR_1\cos\varphi\,dz\,d\varphi,\quad F_y=-2\int_0^{L/2}\int_0^{2\pi} pR_1\sin\varphi\,dz\,d\varphi. \tag{1.3}$$

In calculating the reaction of the gas layer, we considered only the pressure forces. In the approximation chosen to derive the equation for pressure (1.1), these forces are much greater than the frictional forces [2].

The motion of a rotating cylinder under periodically changing external forces in a gravitational field is described the following equations (in the coordinate system connected with the center of the stationary chamber)

$$\begin{aligned} m\ddot{x} &= F_x + m\delta\omega^2\cos\omega t + mg(1 + a_1\cos\omega_1 t),\\ m\ddot{y} &= F_y + m\delta\omega^2\sin\omega t + mga_2\sin\omega_1 t. \end{aligned} \tag{1.4}$$

Here, m is the mass of the rotor; δ is the displacement of the center of mass from the axis of rotation of the rotor; g is acceleration due to gravity; a_1, a_2 are the amplitudes of the

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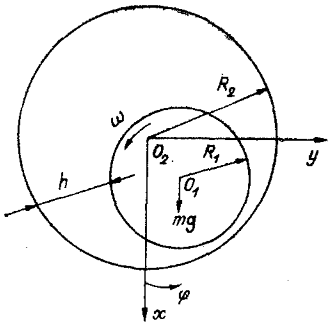

Fig. 1

periodic external forces (such as in the motion of a metal cylinder in a variable electromagnetic field); ω_1 is the frequency of the external forces. At the initial moment of time, the axis of rotation of the rotor coincides with the chamber axis.

2. The problem of the rotation of the rotor will be solved by direct numerical integration of a system of equations describing the motion of the cylinder and the pressure distribution in the gas layer. We rewrite Eq. (1.1) in the form of a conservation law for the volume $R_1\Delta\phi\Delta z h_j$, which includes the node i, j of the grid in the cylindrical coordinate system [3] ($\Delta\phi$ is the angle subdivision of the grid, while Δz is its coordinate subdivision):

$$R_1\Delta\varphi\Delta z\frac{\partial}{\partial t}(h_j\rho_{i,j}) - \frac{h_j^3\Delta\varphi R_1}{12\mu\Delta z}[\rho_{i+1/2,j}(-p_{i,j}+p_{i+1,j}) - \rho_{i-1/2,j}(p_{i,j} - p_{i-1,j})] - \frac{\Delta z}{12\mu R_1\Delta\varphi}[\rho_{i,j+1/2}h_{j+1/2}^3(p_{i,j+1}-p_{i,j}) - \rho_{i,j-1/2}h_{j-1/2}^3(p_{i,j} - p_{i,j-1})] + \frac{\omega R_1\Delta z}{2}(\rho_{i,j+1/2}h_{j+1/2} - \rho_{i,j-1/2}h_{j-1/2}) = 0 \tag{2.1}$$

$$(i = 0,\ 1,\ 2, \ldots,\ I-1,\ j = 1,\ 2,\ \ldots,\ J).$$

Here, $\Delta\phi = 2\pi/J$; $\Delta z = L/2I$; $\phi_j = j\Delta\phi$; $h_j = C - x\cos\phi_j - y\sin\phi_j$; C is the mean size of the radial gap; I and J are the number of grid nodes in the axial and circumferential directions. The relationship between the pressure and density of the gas is given by the ratio p/ρ = const.

In finite-difference form, boundary conditions (1.2) appear as

$$p_{I,j} = p_0,\ p_{-1,j} = p_{1,j},\ p_{i,0} = p_{0,J},\ p_{i,J+1} = p_{i,1} \tag{2.2}$$
$$(i = 1,\ 2,\ \ldots,\ I,\ j = 1,\ 2,\ \ldots,\ J).$$

In writing Eqs. (2.2), we used the conditions of periodicity $p(z, \phi) = p(z, \phi + 2\pi)$ and smoothness $(\partial p/\partial z)_{z=0} = 0$.

We will write Eqs. (2.1) in dimensionless form, using the following units of measurement: distance across the layer C; distance along the layer R_1; time $1/\omega$; pressure p_0:

$$2\Lambda\Delta\varphi\Delta z\frac{\partial}{\partial t}(h_j\rho_{i,j}) - h_j^3\frac{\Delta\varphi}{\Delta z}[\rho_{i+1/2,j}(p_{i+1,j}-p_{i,j}) - \rho_{i-1/2,j}(p_{i,j}-p_{i-1,j})] - \frac{\Delta z}{\Delta\varphi}[\rho_{i,j+1/2}h_{j+1/2}^3(p_{i,j+1}-p_{i,j}) - h_{j-1/2}^3\rho_{i,j-1/2}(p_{i,j}-p_{i,j-1})] + \Lambda\Delta z(\rho_{i,j+1/2}h_{j+1/2} - \rho_{i,j-1/2}h_{j-1/2}) = 0 \tag{2.3}$$
$$(h_j = 1 - x\cos\varphi_j - y\sin\varphi_j).$$

The same notation was used in (2.3) for the dimensionless quantities as for the dimensional quantities.

Using the method of variable directions [4], we reduce the above-formulated problem of calculating the pressure field in the layer to a sequence of unidimensional problems. The coefficients, the nonlinear terms, and the mass flow rates for the gas in the axial direction are referred to the moment of time $t_{k+1/2}$ (where k is the number of the time step), while the remaining quantities are referred to t_k. We approximate the derivative $\partial p/\partial t$ by means of a second-order difference formula centered relative to $t_{k+1/4}$. We solve the resulting nonlinear system of difference equations along the rows of the grid by the trial-run method, regarding the subscript j as a parameter. We determine the initial approximation $p_{i,j}^0$ for each time step by means of linear extrapolation

$$(p_{i,j}^{k+1/2})^0 = 1{,}5p_{i,j}^k - 0{,}5p_{i,j}^{k-1} \quad (k = 1, 2, \ldots),$$

where at the first time step $(p_{i,j}^{1/2})^0 = p_{i,j}^0$.

In the next stage of the method of variable directions, in Eqs. (2.3) we refer the linear components of the mass flow flow rates in the circumferential direction to t_{k+1}, while the coefficients and the remaining terms of the equations are referred to $t_{k+1/2}$. We center the derivative $\partial p/\partial t$ relative to $t_{k+3/4}$. Then employing the method of cyclic trial runs [5], we find the pressure at all of the nodes and the force acting on the rotor from the direction of the gas layer at the moment of time t_{k+1}.

We numerically integrate the equations of motion of the rotor (1.4) by the following scheme:

$$\begin{gathered} x_{k+1/2} = x_{k-1/2} + \dot{x}_k\Delta t,\ y_{k+1/2} = y_{k-1/2} + \dot{y}_k\Delta t, \\ \dot{x}_{k+1/2} = \dot{x}_{k-1/2} + f_{xk}\Delta t,\ \dot{y}_{k+1/2} = \dot{y}_{k-1/2} + f_{yk}\Delta t, \\ x_{k+1} = x_k + \dot{x}_{k+1/2}\Delta t,\ y_{k+1} = y_k + \dot{y}_{k+1/2}\Delta t, \\ \dot{x}_{k+1} = \dot{x}_k + f_{xk+1/2}\Delta t,\ \dot{y}_{k+1} = \dot{y}_k + f_{yk+1/2}\Delta t. \end{gathered} \tag{2.4}$$

Here,

$$\begin{gathered} f_x = \frac{1}{M\Lambda^2}\left[\frac{mg\,(1 + a_1\cos\omega' t)}{p_0R_1^2} - \frac{2}{9}\,\Delta\varphi\Delta z\sum_{i,j=0}^{I,J} S_{i,j}\,\sqrt{p_{i,j}}\cos\varphi_j\right] + \delta'\cos t; \\ f_y = \frac{1}{M\Lambda^2}\left[\frac{mga_2\sin\omega' t}{p_0R_1^2} - \frac{2}{9}\,\Delta\varphi\Delta z\sum_{i,j=0}^{I,J} S_{i,j}\,\sqrt{p_{i,j}}\sin\varphi_j\right] + \delta'\sin t; \\ M = \frac{mp_0}{36\mu^2R_1}\left(\frac{C}{R_1}\right)^5;\ \delta' = \frac{\delta}{C};\ \omega' = \frac{\omega_1}{\omega}; \end{gathered}$$

$S_{i,j}$ is a coefficient in Simpson's cubature formula.

We solve system (2.4) with homogeneous initial conditions:

$$\text{at } t = 0\ \ x_0 = y_0 = 0,\ \dot{x}_0 = \dot{y}_0 = 0, \tag{2.5}$$

where $p_{i,j}^0 = 1$.

At the beginning of the computation, we used the approximation $x_{1/2} = x_0 + \dot{x}_0\Delta t/2$, $y_{1/2} = y_0 + \dot{y}_0\Delta t/2$, $\dot{x}_{1/2} = \dot{x}_0 + f_{x0}\Delta t/2$, $\dot{y}_{1/2} = \dot{y}_0 + f_{y0}\Delta t/2$.

3. We will examine the motion of an unbalanced rotor in a gravitational field without periodic loads ($a_1 = a_2 = 0$). The rotor, located initially at the center of the chamber, later begins to move. Figure 2 shows the formation of the steady-state trajectory of the rotor at a velocity n = 100 rps and $\delta' = 0.3$ — curve 1. Curves 2 and 3 correspond to the steady motion of the rotor with n = 140 and 150 rps. The steady trajectories shown in Figs. 2 and 3 are close to circular.

Let us proceed to the study of the rotor in the presence of assigned periodic external perturbations of finite amplitude. We assign perturbations only along the vertical, i.e., we put $a_1 = 1$, $a_2 = 0$, as well as $\omega_1 = \omega$ (the frequencies of rotation of the rotor coincide with the frequencies of the perturbations). Calculations of the trajectories of the rotor were performed for rotor velocities in the range from 60 to 2500 rps with a change in the relative imbalance δ' from 0.1 to 1 (trajectory 4 in Fig. 2 corresponds to n = 100 rps, δ' = 0.3). The steady-state trajectories of the rotor are close to elliptical. It turns out

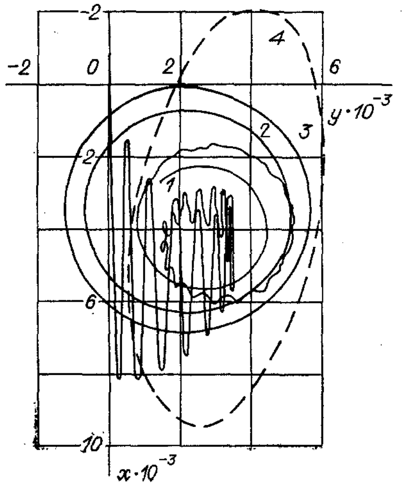

Fig. 2

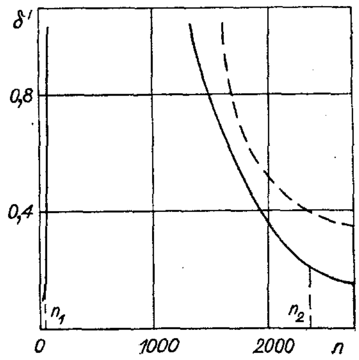

Fig. 3

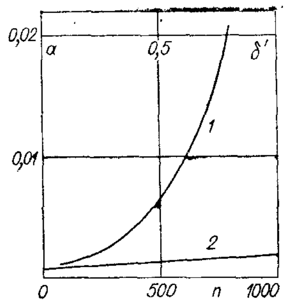

Fig. 4

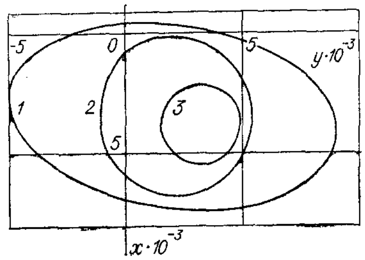

Fig. 5

that at low ($n < n_1$) and high ($n > n_2$) rotational velocities, the motion of the cylinder is unstable: no closed steady-state trajectories of the rotational axis are formed and the rotor touches the chamber. It follows from Fig. 3 that for a given degree of imbalance there is a finite interval of rotor velocities $[n_1, n_2]$ within which the rotor can move without touching the chamber. The left boundary of this interval ($n = n_1$) is nearly independent of the relative imbalance, while a decrease in imbalance displaces the right boundary to the region of higher velocities.

Slight ellipticity of the chamber (the gap function in this case having the form $h_j = 1.066 - 0.133 \cos^2\phi_j - c \cos \phi_j - y \sin \phi_j$) leads to broadening of the zone in which stable elliptical trajectories are formed – due to an increase in the critical rotational velocity n_2. This is illustrated by the dashed curve in Fig. 3.

Figure 4 shows the dependence of the maximum amplitude of the trajectory a on the velocity of the rotor ($\delta' = 0.3$, curve 1). With a tenfold increase in velocity (from 10^2 to 10^3 rps), a increases by a factor of 33.

Figure 5 shows the effect of periodic horizontal perturbations. Curve 1 corresponds to $a_1 = 0$, $a_2 = 1$, $\omega_1 = \omega = 2\pi\ 150\ \text{sec}^{-1}$, while curves 2 and 3 correspond to $a_1 = 0$, $a_2 = 0.1$, $\omega_1 = 1\ \text{sec}^{-1}$, n = 150 and 100 rps ($\delta' = 0.3$). The trajectory of the rotor with the velocity n = 150 rps and $\omega_1 = \omega$ nearly coincides with line 2. An increase in the amplitude of the horizontal perturbations a_2 to 10 in the case of low-frequency disturbances ($\omega_1 = \text{sec}^{-1}$) has almost no effect on the steady circular trajectories of the rotor and does not lead to its destruction. This conclusion is supported by calculations performed for cylinder velocities in the range from 100 to 500 rps. With external perturbations of the amplitude $a_2 = 100$ ($a_1 = 0$, $\omega_1 = 1\ \text{sec}^{-1}$, n = 150 rps/sec), the rotor begins to touch the chamber very soon.

We studied the effect of the degree of imbalance of the rotor on the formation of steady-state trajectories in the presence of periodic horizontal perturbations for $a_1 = 0$, $a_2 = 1$, $\omega_1 = \omega = 200\pi\ \text{sec}^{-1}$. The relative imbalance δ' changed within the range from 0.1 to 1. The amplitude of the steady elliptical trajectories nearly doubled with a change in δ' from 0.2 to 1 (curve 2 in Fig. 4).

A change in the degree of imbalance of the rotor from 0.1 to 0.5 in the presence of vertical perturbations with a frequency equal to half the frequency of rotation of the rotor

($\omega = 200\pi\ \mathrm{sec}^{-1}$, $a_2 = 0$, $a_1 = 1$) had no significant effect on the form or orientation of the rotor's steady-state trajectories. Compared to the case when perturbations are absent (see Fig. 2), the presence of finite vertical perturbations leads to an increase in the amplitude of the steady-state trajectories by a factor of 8-9, a shift in the center of the trajectory from the first to the third quadrant in the (x, y) plane, and transformation of the originally circular path into an elliptical path. Most of our computer calculations were performed on 16×40 and 24×60 grids.

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MOTION OF A BED LOAD

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This article deals with the problem of mathematically modeling a thin moving layer of a two-phase mixture bounded below by a stationary granular medium and above by a liquid flow. The position of the top boundary of the moving layer is specified beforehand along with the normal and shear stresses on it. These characteristics can be obtained from the solution of hydrodynamic equations.

The moving mixture is assumed to be uniform and its acceleration small (the latter is ignored). As is customary in calculations performed for shallow water [1], the contribution of shear stresses on areas normal to the surface is considered to be negligible, while pressure is distributed in accordance with a hydrostatic law.

Formulation of the Problem. In accordance with the above assumptions, the equations of motion are written in the form

$$\begin{gathered}\partial p/\partial s+\rho g\partial\xi/\partial s+\partial\tau_s/\partial m=0,\\ \partial p/\partial l+\rho g\partial\xi/\partial l+\partial\tau_l/\partial m=0,\ \partial p/\partial m=\rho g\cos\gamma,\end{gathered}\tag{1}$$

where $\xi = \xi(x, y)$ is the equation of the surface of the mixture; x and y are horizontal cartesian coordinates; s and ℓ are orthogonal curvilinear coordinates on the surface of the mixture; m is the axis directed along a normal to the surface of the mixture (m = 0 on the surface $\xi = \xi(x, y)$); p is the pressure in the mixture; τ_s and τ_ℓ are projections of the shear stress $\boldsymbol{\tau}$ on areas parallel to the surface of the mixture; ρ is the density of the mixture ($\rho = f\rho_r + (1 - f)\rho_b$); ρ_r and ρ_b is the density of the particles and water; f is concentration, the value of which is determined below; γ is the acute angle between the normal to the surface of the mixture and a vertical line.

The rheological relation for the shear stress includes Coulomb's law for a bulk medium and Prandtl's law for a fluid:

$$\boldsymbol{\tau}=-\left(\frac{\partial\mathbf{u}}{\partial m}\Big/\left|\frac{\partial\mathbf{u}}{\partial m}\right|\right)(p_s\operatorname{tg}\varphi+\tau_b).\tag{2}$$

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